Origami Sekkei: A Mathematical Approach to the Ancient Art of Paper Folding

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Spring 2019

Contact Information

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Course Description

The art of paper folding has existed for over fifteen centuries, yet an astonishing 98 percent of new origami designs have been developed within the past fifty years and with rapidly increasing complexity. This modern day origami renaissance has been closely connected to advances in science, technology, and computational mathematics combined with artistic intuition and creativity. **How can mathematical methods be applied to develop awesome, creative origami with a purpose?** The practice of origami sekkei (折り紙設計), or technical origami design, overthrows the traditional freestyle folding process and instead turns to a carefully engineered theoretical model. Students will explore contributions from the pioneers of modern origami, ranging from the fantastically intricate work of NASA physicist Robert J. Lang to the elegant simplicity of origami grandmaster Kiyo Yoshizawa. The course will cover mathematical techniques such as base folding, grafting, circle packing, tree theory, and box pleating through hands-on, interactive exercises. Starting with the very basics, this course is designed to be approachable to beginners in all aspects but also offers topics that may be of additional interest for those in specialized fields of study. Overall, we hope to unfold the mysteries of origami and turn the page to reveal some of the most cutting-edge work in the field.

Course Objectives and Learning Outcomes

In this course, students will explore how to create innovative origami. Unlike a fixed step-by-step process, this class is designed to introduce an array of mathematical tools and design tricks to provide a basis for creation. By the end of the semester, students will feel confident to:

- Demonstrate comprehension of fundamental mathematical principles through breaking down origami designs into their basic structures.
- Replicate existing origami designs, and modify these designs into new reformulations.
- Implement technical methodologies to formulate an original origami artwork, starting with a theoretical tree model, using this model to design a crease pattern, and then finally folding a physical paper design.

Optional Texts, Materials, and Required Reading

There are no required texts and materials will be provided. Required Reading will be available on the Canvas website as a PDF.

Optional texts featured in this course include "Origami Design Secrets: Mathematical Methods for an Ancient Art" by Robert J. Lang and "Origami ^{5} : Fifth International Conference on Origami in Science, Mathematics, and Education", edited by Patsy Wang-Iverson. Some of my favorite designs are detailed in "Akri Yoshizawa: Japan's Greatest Origami Master" and "The Complete Book of Origami: Step-by-Step Instructions in Over 1000 Diagrams".

Assignments

Engaged Participation in Class: This class will involve hands-on exercises in almost every session, so engaged in-class participation is very important. While not required for the completion of this course, I would also recommend review of materials posted on Canvas, especially for unfamiliar topics.

Weekly Assignment: Students must create at least one origami creation to share with the class each week, either using an existing or new design. There will be a large repository of origami resources, including tutorials, posted to the Canvas portal. However, students may feel free to create or replicate any design of their choosing in order to further solidify familiarity with their folding skills and find inspiration in prior artwork. Please upload a photo of your design to Canvas by Tuesday night!

Origami Theory Analysis: Students will complete a completion graded worksheet to break down various origami designs into their underlying mathematical structures. This exercise will occur around midway through the semester and focus on lessons in tiling, circle packing, and tree theory. The goal of this exercise is to demonstrate comprehension of these fundamental mathematical principles.

Final Project: The semester will end with an open ended final project to design and present a novel origami artwork using class concepts. Students will be encouraged to focus not only on the folding pattern itself, but also the artistic presentation of this design. The objective of this project is to showcase your new origami skills!

Grade Policies

Students will receive a grade of Satisfactory (S) or Unsatisfactory (U) based on the following rubric:

20% - Class Attendance and Participation
50% - Weekly Folding Assignment
10% - Origami Theory Analysis
20% - Final Project

Absence Policies

Students may have three absences, excused and unexcused combined. Please let me know in advance that you won't be able to come to class. Any more than three absences will result in a letter grade reduction.

Rice Honor Code

In this course, all students will be held to the standards of the Rice Honor Code, a code that you pledged to honor when you matriculated at this institution. If you are unfamiliar with the details of this code and how it is administered, you should consult the Honor System Handbook at http://honor.rice.edu/honor-system-handbook/. This handbook outlines the University's expectations for the integrity of your academic work, the procedures for resolving alleged violations of those expectations, and the rights and responsibilities of students and faculty members throughout the process.

Disability Resource Center

If you have a documented disability or other condition that may affect academic performance you should: 1) make sure this documentation is on file with the Disability Resource Center (Allen Center, Room 111 / adarice@rice.edu / x5841) to determine the accommodations you need; and 2) talk with me to discuss your accommodation needs.

Syllabus Change Policy

This syllabus is only a guide for the course and is subject to change with advanced notice.

Course Schedule

January 15, 2019 - Introduction

- Course overview
- Key terminology (symbols, terms, and diagrams) through a step-by-step origami folding example.
- Brief history of origami innovations, from ancient origins to modern day technical folding.

January 22, 2019 - Base Folding Strategy

- Traditional Kite, Fish, Frog, and Bird Base and the corresponding offset variants as the most basic methodologies for origami construction.
- Contrast uniaxial bases with multiaxial bases, such as the Windmill base and Montroll's base.
- Exercise: How do you decide which base to use for a given structure? How does the relationship between the four classic bases reflect some underlying origami structure?

January 29, 2019 - Splitting Points

- Easy techniques for splitting points: Yoshizawa split, Montroll split, optimal length split.
- Calculation for efficient point splitting.
- Exercise: Classifying subjects by the number of significant points, and practice implementation by designing and folding an increasingly complicated elephant.

February 5, 2019 - Introduction to Grafting

- Border grafting: attaching smaller base squares to the perimeter of the larger square and transitioning from a two-sheet design into a one-sheet design.
- Strip grafting: more powerful technique to maintain the same proportions of the grafted design with the original design.
- **Required Reading:** Grafting (Lang 2011), pages 129 - 145 (PDF provided).
- Exercise: Songbird model example from Robert J. Lang

February 12, 2019 - Tessellations, Textures, and Pattern Grafting

- Adding patterned or textured surfaces to existing designs using pleat grafting (turtle shell pattern example).
- Exercise: Calculation of space allocation for different pleat textures, then implementation.

February 19, 2019 - Tiling and Stick Figure Notation

- Definition of a single tile and stick figure notation.
- Concatenation rules for multiple tiles through aligning "circles" and "rivers". Essentially, we want to make a structure that topologically resembles a graph theoretic weighted tree.
- **Required Reading:** Excerpts from Chapters 8-10 of Lang 2011 (PDF provided).
- Exercise: Understanding origami structure through dissecting existing designs into their constituent components.

February 26, 2019 - Circle Packing

- Introduction to Circle Packing: If you know the design you want, how do you arrange your circles?
- How tiling concepts connect to the overall optimization problem of circle packing.
- Principles relating circle arrangement to crease patterns.
- Problem of packing N non-overlapping circles into a square (this turns out to be an NP-hard problem but we can use an approximation that is good enough for our purposes)

March 4, 2019 - Introduction to Tree Theory

- This is the first robust algorithm for origami design (Lang and Meguro).
- Transition from stick figure notation to the full crease pattern. Since this builds off the previous two lectures, it may be necessary to briefly review.
- **Required Reading:** Section 5 of TreeMaker - a program for Origami Design (pg. 112-143). https://www.langorigami.com/wp-content/uploads/2015/09/TreeMkr40.pdf
- Exercise: Tree Theory worksheet in class.

March 11, 2019 - Computational Tree making using TreeMaker software

- Download and learn TreeMaker software
- In class exercise to implement computational tools in an origami design.
- **Required Reading:** Section 2.1 of TreeMaker - a program for Origami Design. https://www.langorigami.com/wp-content/uploads/2015/09/TreeMkr40.pdf

March 18, 2019 - NO CLASS

March 25, 2019 - Universal and Freeform Origami

- Aviv Ovadya's surface extrusions, and Erik D Demaine's Maze Folding algorithm provide universal hinge pattern.
- Contrast 1D Tree-Based methods (e.g. TreeMaker Software) with 3D Freeform Origami (e.g. Origamizer) methods.

April 1, 2019 - Box Pleating

- Transition from radial folding (common using previously explored methods in tree theory) to a matrix of parallel folds.
- This method is technically more restrictive, but allows for easier implementation of grafting and making new flaps.
- Exercise: Implementing techniques for box folding and joining multiple boxes.

April 8, 2019 - Mechanical Properties of Folded Materials

- Examples of structural origami such as the Miura Ori Fold and applications in engineering.
- Programmable materials that can change structural properties by introducing point defects.
- Applications of computational origami in robotics (minimally invasive surgery) and space (StarShade mission, foldable satellites).
- Exercise: Modify the Miura Ori Fold structure to yield different mechanical properties in an origami-based "material".

April 15, 2019 - Work day for Final Project

- In class work day

April 22, 2019 - Present Final Project

- In class final presentation